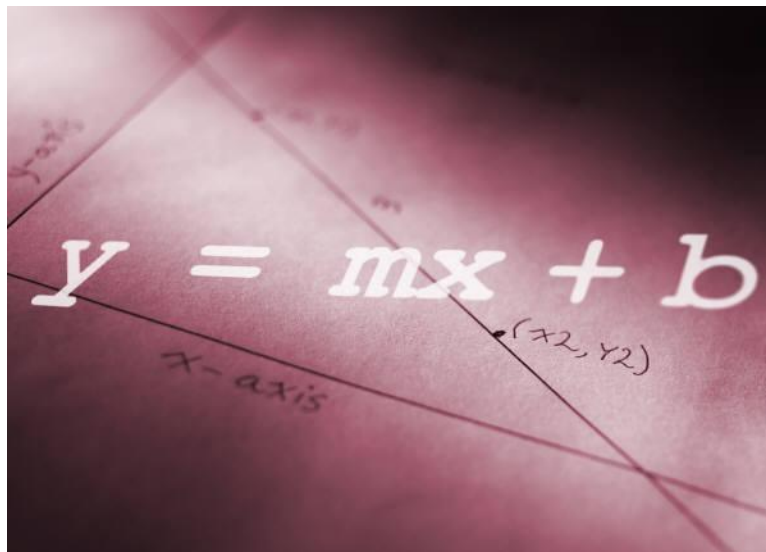
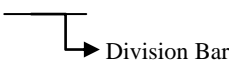


Algebra I Study Guide

**By Mary Shoemaker
2008**



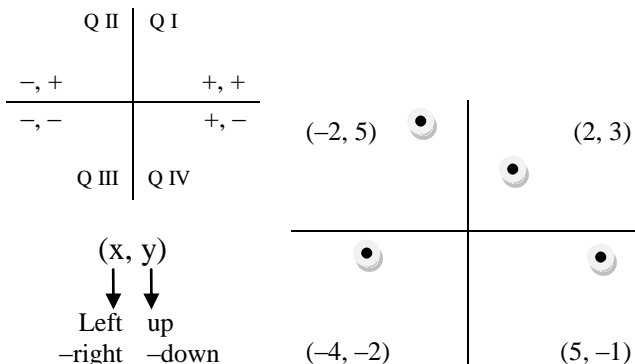
Order of Operations

1. Parentheses: inside out [], (), {},  Division Bar
2. Exponents
3. Multiply or divide: Left → Right
4. Add or subtract Left: → Right

Examples

$$\begin{array}{ccc} 3 - 12 \div 4 & 6^2 \div 4 \cdot 3 & 4^2 \cdot 2 + [7 - (3^2 - 5)] \\ 3 - 3 & 36 \div 4 \cdot 3 & 16 \cdot 2 + [7 - (3^2 - 5)] \\ 0 & 9 \cdot 3 & 32 + [7 - 4] \\ & & 35 \end{array}$$

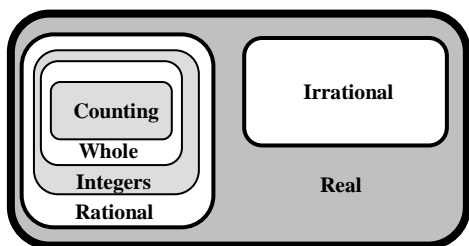
Coordinate Graphing



Real Numbers

- **Natural or Counting Numbers** : $N \in \{1, 2, 3, 4, \dots\}$
- **Whole Numbers**: $W \in \{0, 1, 2, 3, 4, \dots\}$
- **Integers**: $I \in \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- **Rational Numbers (R)**: Any number that can be written as a fraction a/b . *Natural, whole, and integers are rational.*
- **Irrational Number (Q)**: Numbers that CANNOT be written as a fraction, like non-repeating decimals and radicals
- **Real Numbers (R)**: All rational and irrational numbers

Numbers are either rational or irrational



Integers

How to Add Integers

- **Same signs**: Add and keep the sign
- **Different signs**: Subtract large – small
Keep sign of the larger number

How to Subtract Integers

- Change all subtraction problems to addition
 $a - b = a + (-b)$ $a - (-b) = a + b$
- Follow rules for addition of integers

How to Multiply and Divide Integers

- **Two Numbers**
Same signs → Positive
Different signs → Negative
- **More than two Numbers**
Odd → Negative
Even → Positive

Properties

- **Commutative Property of Addition/Multiplication**
 $a + b = b + a$ $ab = ba$
- **Associative Property of Addition/Multiplication**
 $(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$
- **Distributive Property**
 $a(b + c) = ab + ac$
- **Identity Properties**
 $a + 0 = a$ and $0 + a = a$ $a \cdot 1 = a$ and $1 \cdot a = a$
- **Inverse Properties**
 $a + -a = 0$ $a \cdot \frac{1}{a} = 1$

The number $\frac{1}{a}$ is called the **reciprocal** or **multiplicative inverse**.

- **Properties of 0**
 $a \cdot 0 = 0$ and $0 \cdot a = 0$
 $0/a = 0$ where $a \neq 0$
Division by 0 is undefined

Add-Subtract-Multiply-Divide Polynomials

Adding and Subtracting

- Combine like terms only
- Rewrite subtraction problems
- Watch SIGNS!

Examples

$$\begin{array}{r} (3x + 4) + (2x - 1) \\ 5x + 3 \end{array}$$

$$\begin{array}{r} (4d - 2) - (5d - 3) \\ 4d - 2 - 5d + 3 \\ -d + 1 \end{array}$$

$$\begin{array}{r} (x^2 + x + 1) + (2x^2 + 3x + 2) \\ 3x^2 + 4x + 3 \end{array}$$

$$\begin{array}{r} (4x^2 + 3x + 2) - (2x^2 - 3x + 7) \\ 4x^2 + 3x + 2 - 2x^2 + 3x - 7 \\ 2x^2 + 6x - 5 \end{array}$$

$$2x(5x - 4) = 10x^2 - 8x$$

$$-2(5a - 4) = -10a + 8$$

$$\frac{2x^2 + 6}{2} = x^2 + 3$$

$$\frac{20c + 8}{4} = 5c + 2$$

Equations

$3x = 27 \Rightarrow$ Divide for answer $\Rightarrow x = 9$
 $x = 18 \Rightarrow$ Multiply for answer $\Rightarrow x = 54$
 $\frac{\quad}{3}$

Two-Step Equations

Solve in reverse order of operations

- $+/-$ first
- then x/\div

Watch for negatives

$-7 - 13y = 32$	$\frac{x + 5}{4} = 12$
$-13y = 39$	$\frac{x}{4} = 7$
$y = -3$	$x = 28$

Variables on Both Sides of Equation

Move the smaller variable by adding the opposite to both sides of the equation. (*look to move negative variables*)

$$\begin{aligned} (+3y) 5y - 10 &= 14 - 3y (+3y) \\ 8y - 10 &= 14 \\ 8y &= 24 \\ y &= 3 \end{aligned}$$

More Equations

Combine Like Variables

$$\begin{aligned} 4n - 2 + 7n &= 20 \\ 11n - 2 &= 20 \\ 11n &= 22 \\ n &= 2 \end{aligned}$$

Distribute to Eliminate Parentheses

$$\begin{aligned} 3(r - 4) &= 9 \\ 3r - 12 &= 9 \\ 3r &= 21 \\ r &= 7 \end{aligned}$$

Multi-Step Equations

1. Distribute
2. Combine
3. Move variables to the same side of the equation
4. Solve remaining equation

*Big Tip: Look to get rid of fractions on variables.
Multiply all terms by the common denominator.*

More Equations

Multi-Step Example

$$\begin{aligned} 3x - 2(x + 6) &= 4x - (x - 10) \\ 3x - 2x - 12 &= 4x - x + 10 \\ x - 12 &= 3x + 10 \\ -12 &= 2x + 10 \\ -22 &= 2x \\ -11 &= x \end{aligned}$$

Literal Equations

Treat variables like numbers...

$T + M = R$, solve for T	$A = lw$, solve for w
$T = R - M$	$\frac{A}{l} = w$

$ax + r = 7$, solve for x

$$\begin{aligned} ax &= 7 - r \\ x &= \frac{7 - r}{a} \end{aligned}$$

Proportions

Proportion: An equation showing that two ratios are equal.

Cross Products are equal

$\frac{a}{b} = \frac{c}{d}$	$a \cdot d = b \cdot c$
$\frac{3}{4} = \frac{9}{12}$	$4 \cdot 9 = 3 \cdot 12$ $36 = 36$

Examples

$\frac{15}{m} = \frac{10}{9}$	$10m = 135$	$m = 13.5$
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9 	25 	$\frac{9}{27} = \frac{25}{x}$
27	x	$9x = 675$ $x = 75$

Percent Problems

Converting Percents

- **P** \Rightarrow **D**: Move decimal two to the left (2 \leftarrow)
- **D** \Rightarrow **P**: Move decimal two to the right (2 \rightarrow)
- **P** \Rightarrow **F**: Write number over 100 and reduce
Hint: Write percents with one decimal over 1000, two decimals of 10,000, etc.
- **F** \Rightarrow **D**: Divide numerator by denominator

Percent Problems using Equations

- Write the equation as you read the problem.
 is \Rightarrow = of \Rightarrow x what \Rightarrow x into, out of \Rightarrow \div
- Write percents as decimals or fraction

Percent Change

$\text{Percent} = \frac{\text{change}}{\text{original}}$	<i>Subtract amounts and divide by original</i>
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Test Taking Tips

- Skip problems that you don't know how to do and come back.
- Write down all your steps. In other words, don't do the work in your head.
- When you panic, stop, and do something that relaxes you, like close your eyes and take a deep breath.
- If you finish early, go back and do the problems again. Don't look at your previous work.

Consistent/Inconsistent Systems

Inconsistent System: No solution, Lines are parallel

$$\begin{aligned} 2x - y &= -1 \\ 4x - 2y &= 4 \\ 0 &\neq -6 \end{aligned}$$

Both lines have the same slope

Consistent System:

- Independent: lines intersect, One solution
- Dependent: Infinitely many solutions, Same line!

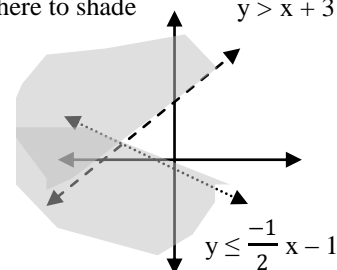
$$\begin{aligned} x - y &= 2 \\ 2x - 2y &= 4 \\ 0 &= 0 \end{aligned}$$

Second equation is 2x the first equation

Graphing Inequality Systems

$> <$ \leftarrow \leftarrow \leftarrow Type of line
 $\geq \leq$ \leftarrow \leftarrow \leftarrow Type of line

$> \geq$ above \leftarrow Where to shade
 $< \leq$ below \leftarrow Where to shade



Coin Problems/Age Problems

Coins

Usually involved with number of coins and monetary value

A coin bank has 250 dimes and quarters worth \$39.25 How many dimes and quarters are there?

$$d + q = 250 \quad .10d + .25q = 39.25$$

Age

Represent a person's age in the past, present, or future

A father is 32 years older than his son. In four years, the father will be 5 times older. How old are they now?

$$\begin{aligned} s + 32 &= f \leftarrow (\text{age now}) \\ f + 4 &= s + 4 \leftarrow (\text{age in 4 years}) \\ f + 4 &= 5(s + 4) \end{aligned}$$

Wind/Current and Digit Problems

Wind/Current

Need speed when traveling with the wind and against the wind

Use $r \cdot t = d$

$$(r + w)2.5 = 750$$

$$(r - w)2 = 750$$

r - rate; w = wind

Digit

Write value of number in expanded form from (t: tens; u: ones)

- Two digit number: $10t + u$
- Reverse digits: $10u + t$
- Sum of digit: $t + u$

Laws of Exponents

Product of Powers: $x^m \cdot x^n = x^{m+n}$ $x^6 \cdot x^4 = x^{10}$

Power of Power Property: $(x^m)^n = x^{m \cdot n}$ $(x^6)^4 = x^{24}$

Power of a Product: $(xy)^m = x^m y^m$ $(xy)^5 = x^5 y^5$

Hint: even powers \rightarrow Positive answer
odd powers \rightarrow Negative answer

$m > n$ $\frac{x^m}{x^n} = x^{m-n}$ $\frac{x^6}{x^4} = x^2$

$m < n$ $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$ $\frac{x^4}{x^6} = \frac{1}{x^2}$

Power of Fractions:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$$

$$x^{-n} = \frac{1}{x^n} \quad x^{-6} = \frac{1}{x^6}$$

$$x^0 = 1 \quad 5^0 = 1$$

Scientific Notation

A number written as a product with two factors:

- A number between 0 and 9 and
- A power of 10

Examples

$$463,000,000 = 4.63 \times 10^8$$

$$.000597 = 5.97 \times 10^{-4}$$

Move right for a negative exponent; left for positive

Count the number of times you need to move the decimal to make a number less than 10

$$(3 \times 10^3)(4 \times 10^{-5})$$

$$12 \times 10^{-2}$$

$$1.2 \times 10^{-1}$$

$$\frac{2.5 \times 10^6}{5 \times 10^2}$$

$$.5 \times 10^4$$

$$5 \times 10^3$$

Add exponents

Subtract exponents

Special Products

Perfect Squares

- $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
- $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Difference of Squares

- $a^2 - b^2 = (a + b)(a - b)$

Examples

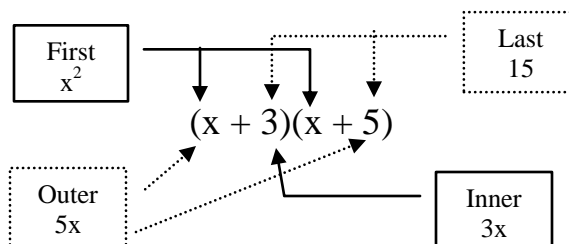
$$\begin{aligned} (x + 4)^2 \\ x^2 + 8x + 16 \end{aligned}$$

$$\begin{aligned} (2x - 3)^2 \\ 4x^2 - 6x + 9 \end{aligned}$$

$$\begin{aligned} (a + 4)(a - 4) \\ a^2 - 4 \end{aligned}$$

$$\begin{aligned} (3r + 4)(3r - 4) \\ 9r^2 - 16 \end{aligned}$$

FOIL: (First, Outer, Inner, Last)



Examples

$$\begin{aligned} (3x + 2)(5x + 1) \\ 15x^2 + 3x + 10x + 2 \\ 15x^2 + 13x + 2 \end{aligned}$$

$$\begin{aligned} (x + 9)(2x - 4) \\ 2x^2 - 4x + 18x - 36 \\ 2x^2 + 14x - 36 \end{aligned}$$

Factoring

Try to factor in this order:

- **GCF:** Take out numbers and letters that are common to all
 $4x^2 + 12x = 4x(x + 3)$
- **Factor by grouping:** Terms in both parentheses should be the same

$$\begin{aligned} x^2 + x + 4x + 4 \\ x(x + 1) + 4(x + 1) \\ (x + 4)(x + 1) \end{aligned}$$

- **Perfect Squares:** 3 terms, first and last terms are squares

$$\begin{aligned} x^2 + 8x + 16 \\ (x + 4)^2 \end{aligned}$$

- **Difference of Squares:** 2 terms, both squares

$$\begin{aligned} x^2 - 25 \\ (x + 5)(x - 5) \end{aligned}$$

- **Trial and Error:** Next box

Factoring: Trial and Error

Look at signs

+	+	→	(+)(+)
-	+	→	(-)(-)
+	-	→	(+)(-)
-	-	→	(+)(-)

Think of factors for last number and sums or differences of those factors to find the answer...

$$\begin{aligned} x^2 + 6x + 8 \\ (x + 2)(x + 4) \end{aligned}$$

Examples

$$\begin{aligned} x^2 + 7x + 12 \\ (x + 4)(x + 3) \end{aligned}$$

$$\begin{aligned} x^2 - 7x + 12 \\ (x - 4)(x - 3) \end{aligned}$$

$$\begin{aligned} x^2 + x - 12 \\ (x + 4)(x - 3) \end{aligned}$$

$$\begin{aligned} 5x^3 + 50x^2 + 45x &= 5x(x^2 - 10x + 9) \\ &= 5x(x - 9)(x - 1) \end{aligned}$$

Finding the Zeroes

Tells where a parabola crosses the x-axis.

Examples

$$\begin{aligned} y = x^2 + 8x - 9 \\ (x + 9)(x - 1) \\ x = -9 \text{ or } x = 1 \end{aligned}$$

$$\begin{aligned} y = x^2 - 4 \\ (x + 2)(x - 2) \\ x = -2 \text{ or } x = 2 \end{aligned}$$

No Solution

- Cannot factor
- Does not cross the x-axis

$$y = x^2 + 4$$

One Solution

- Perfect Squares
- Sits on x-axis

$$\begin{aligned} y = x^2 - 10x + 25 \\ (x - 5)^2 \\ x = 5 \end{aligned}$$

Quadratic Function: Parabolas

$$y = ax^2 + bx + c$$

Vertex form:

$$y = a(x - h)^2 + k$$

Direction/Steepness

Vertical

horizontal shift

- **Steepness:** $> 1 \rightarrow$ fat; $< 1 \rightarrow$ skinny
- **Vertical shift:** positive \rightarrow up; negative \rightarrow down
- **Horizontal shift:** positive: left; negative: right
- **Vertex:** lowest (or highest point), minimum (maximum)
- **Axis of symmetry:** vertical line through the vertex ($x = 3$)

Examples

$$\begin{aligned} y = (x - 3)^2 - 3 \\ \text{vertex: } (3, -3) \end{aligned}$$

$$\begin{aligned} y = -(x + 2)^2 + 4 \\ \text{vertex: } (-2, 4) \end{aligned}$$

Solving Equations by Square Roots

Look for squares!

Solve by taking the square root of both sides of the equation.

$$x^2 = 25 \qquad x^2 = \frac{9}{4} \qquad (x-2)^2 - 9 = 0$$

$$x = \pm 5 \qquad x = \pm \frac{3}{2} \qquad (x-2)^2 = 9$$

$$x - 2 = \pm 3$$

$$x = 5 \text{ or } x = -1$$

$$25t^2 - 144 = 0 \qquad (x-1)^2 - 11 = 0$$

$$25t^2 = 144 \qquad (x-1)^2 = 11$$

$$t^2 = \frac{144}{25} \qquad x - 1 = \pm \sqrt{11}$$

$$t = \pm \frac{12}{5} \qquad x = 1 \pm \sqrt{11}$$

Completing the Square

$$x^2 + bx + \left(\frac{b}{2}\right)^2$$

half of b, then squared

Use to solve quadratic equations or to write quadratic equations in vertex form.

Steps to complete the square

- Group $x^2 + bx$
- Complete the Square
- Add and subtract the "square"
- Factor the "perfect square"
- Subtract the two numbers

$$y = x^2 - 8x + 7$$

$$\left(\frac{-8}{2}\right)^2 = 16$$

$$(x^2 - 8x + 16) + 7 - 16$$

$$(x - 4)^2 + 7 - 16$$

$$y = (x - 4)^2 - 9$$

Quadratic Equations

Try to solve equations by:

- Taking the square root
- Factoring
- Completing the square
- Using the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

$$3x^2 - 10x + 7$$

$a = 3; b = -10; c = 7$

$$x = \frac{10 \pm \sqrt{10^2 - 4(3)(7)}}{2(3)} = \frac{10 \pm \sqrt{16}}{6} = \frac{10 \pm 4}{6} = 1 \text{ or } \frac{7}{3}$$

Discriminant/Quadratic Inequalities

Discriminant: $b^2 - 4ac$

- if $< 0 \rightarrow$ no solution (parabola is above x-axis)
- if $= 0 \rightarrow$ one solution (parabola sits on x-axis)
- if $> 0 \rightarrow$ two solutions (parabola goes thru x-axis at 2 points)

Graphing Quadratic Inequalities

Type of line

- $< >$ dotted
- $\leq \geq$ solid

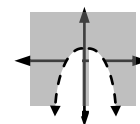
Shading test

- Choose point on the axis of symmetry and do true/false test

Example

$$y > -x^2 + 1$$

inside point; (0, 0)
 $0 > -0^2 + 1$
 $0 > 1$, False
 Shade outside



Rational Functions

Domain: All real numbers that give a nonzero denominator.
 When the denominator is 0, the function is undefined

Asymptote: A vertical or horizontal line that the graph approaches but does not cross.

Graphing Rational Functions

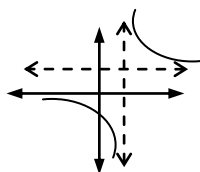
$$y = \frac{1}{x-h} + k$$

Direction

Horizontal shift

Vertical shift

$$y = \frac{1}{x-2} + 3$$



Simplify Rational Expressions

Steps to simplify

- Factor
- State restrictions
- Cancel (like a fraction)

Works for multiplying and dividing rational expressions!

Examples

$$\frac{3x-6}{x^2+x-6} = \frac{3(x-2)}{(x+3)(x-2)} = \frac{3}{x+3} \quad x \neq -3; x \neq 2$$

$$\frac{x^2+x-20}{16-x^2} = \frac{(x+5)(x-4)}{(4-x)(4+x)} = \frac{-(x+5)}{(x+4)} \quad x \neq -4; x \neq 4$$

$$\frac{x^2+6x+8}{x^2-4x+3} \cdot \frac{x^2+x-4}{5x+10} = \frac{(x+4)(x+2)}{(x-3)(x-1)} \cdot \frac{(x+3)(x-1)}{5(x+2)}$$

$$= \frac{(x+4)(x+3)}{5(x-3)} \quad x \neq 3; x \neq 1; x \neq -2$$

Add/Subtract Rational Expressions

Steps to Simplify

- Factor denominator
- State restrictions
- Find a common denominator
- Multiply each numerator by the missing factor
- Simplify numerator

Examples

$$\frac{x}{x^2-1} + \frac{2}{x+1} = \frac{x+2(x-1)}{(x+1)(x-1)} = \frac{3x-2}{x^2-1}$$

$$\frac{5x+4}{x-2} - \frac{7+3x}{x-2} = \frac{5x+4-7-3x}{x-2} = \frac{2x-3}{x-2}$$

Watch negative

Notice change in sign

Solving Rational Equations

Steps to Solve Rational Equations

- Factor
- State restrictions
- Find a common denominator
- Multiply each numerator by the missing denominator
- Solve the equation (if you have x^2 , you need to factor)

Example

$$\frac{2}{x-1} + \frac{3}{x+1} = 1$$

Common denominator
(x-1)(x+1)

$$2(x+1) + 3(x-1) = x^2 - 1$$

$$2x + 2 + 3x - 3 = x^2 - 1$$

$$5x - 1 = x^2 - 1$$

$$x \neq -1; x \neq 1$$

$$0 = x^2 - 5x$$

$$0 = x(x-5)$$

$$x = 0 \text{ or } x = 5$$

Operations with Radicals

Solve for Simplest Radical Form

$$\frac{\sqrt{800}}{\sqrt{400x^2}} = 20\sqrt{2}$$

$$(5\sqrt{3})^2 = 25 \cdot 3 = 75$$

$$\sqrt{3} \cdot \sqrt{6} = \sqrt{18}$$

$$\sqrt{2 \cdot 9} = 3\sqrt{2}$$

$$\sqrt{2}(6 + \sqrt{12}) = 6\sqrt{2} + \sqrt{24} = 6\sqrt{2} + \sqrt{4 \cdot 6} = 6\sqrt{2} + 2\sqrt{6}$$

$$(3 - \sqrt{2})(4 + \sqrt{2}) = 12 + 3\sqrt{2} - 4\sqrt{2} - 2 = 10 - \sqrt{2}$$

F O I L

Divide exponent by 2; remainder in radical

More Operations with Radicals

$$\frac{\sqrt{50} + 3\sqrt{72}}{\sqrt{25x^2} + 3\sqrt{36x^2}} = \frac{5\sqrt{2} + 18\sqrt{2}}{23\sqrt{2}}$$

You *cannot* leave a radical in the denominator

- Simplify by multiplying the numerator and denominator by the radical that is in the denominator

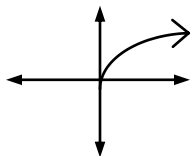
$$\sqrt{\frac{9}{5}} \cdot \sqrt{\frac{5}{5}} = \frac{\sqrt{45}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$$

$$\sqrt{\frac{x^4}{y^5}} = \frac{x^2}{y^2\sqrt{y}} \cdot \sqrt{\frac{y}{y}} = \frac{x^2\sqrt{y}}{y^3}$$

Square Root Function

Graph

- $y = \sqrt{x}$
- domain: $x \geq 0$
- range: $y \geq 0$



Solving Equations

- Square each side
- Check your answer!

Square both sides

Examples

$$\sqrt{x+2} = 3$$

$$x+2 = 9$$

$$x = 7$$

$$\sqrt{3x+4} = 5$$

$$3x+4 = 25$$

$$3x = 21$$

$$x = 7$$

$$\sqrt{2x+6} = x-1$$

$$2x+6 = x^2-2x+1$$

$$0 = x^2-4x-5$$

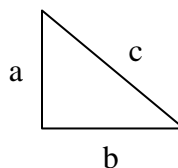
$$0 = (x-5)(x+1)$$

$$x = 5 \text{ or } x = -1$$

$$\sqrt{2(5)+6} = 5-1; 4=4$$

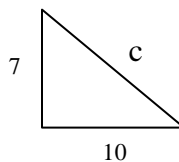
$$\sqrt{2(-1)+6} = -1-1; \sqrt{4} \neq -2$$

Pythagorean Theory



$$a^2 + b^2 = c^2$$

Examples

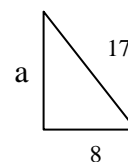


$$7^2 + 10^2 = c^2$$

$$49 + 100 = c^2$$

$$149 = c^2$$

$$c = 12.2$$



$$a^2 + 8^2 = 17^2$$

$$a^2 + 64 = 289$$

$$a^2 = 225$$

$$a = 15$$

Distance & Midpoint

Distance Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Examples

(3, 2); (11, 8)

Distance

$$\sqrt{(11-3)^2 + (8-2)^2}$$

$$\sqrt{(8)^2 + (6)^2}$$

$$\sqrt{64 + 36}$$

$$\sqrt{100}$$

$$10$$

Midpoint

$$\left(\frac{3+11}{2}, \frac{2+8}{2}\right)$$

$$\left(\frac{14}{2}, \frac{10}{2}\right)$$

$$(7, 5)$$

Matrices

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

entry name: [row, column]

Examples

Addition/Subtraction: Do operation with corresponding entries

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Scalar Multiplication: Multiply each entry by factor

$$3 \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix}$$

Parent Functions & Transformations

$y = mx + b \rightarrow$ Linear ↗

$y = a|x-h| + k \rightarrow$ Absolute Value ↘

$y = a^x \rightarrow$ Exponential ↗

$y = a(x-h)^2 + k \rightarrow$ Quadratic ↶ ↷

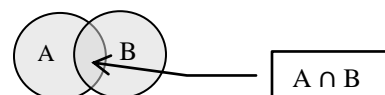
$y = \frac{a}{x-h} + k \rightarrow$ Rational ↷ ↶

$y = a\sqrt{x} \rightarrow$ Square Root (Radical) ↗

h: Horizontal Shift; k: vertical shift; a: stretch; m: slope

A negative before the a indicates upside down direction

Sets & Counting



Venn diagram: Circles used to represent sets

Inter(\cap): Overlapping region when two sets contain common items

Union (U): Items from both sets

Counting elements of sets: Two sets combined minus the intersection

Fundamental Counting Principal

First choice: m ways

Second choice: n ways

$m \cdot n = \#$ of ways to make combinations

Example

20 styles of shoes; 4 sizes, 3 colors. How many different shoes?

$$20 \cdot 4 \cdot 3 = 240 \text{ shoes}$$

Probability

Probability: Likelihood that an event will occur

Experimental probability: Number of outcomes that an event occurred during an experiment

$$P(E) = \frac{\# \text{ of outcomes}}{\text{total number of trials}}$$

Theoretical probability: Number of favorable outcomes found in a sample space (determined without an experiment)

Sample space: Set of all possible outcomes

Examples

A number cube was rolled 100 times. An even number appeared 47 times. What was the experimental probability? What is the theoretical probability?

$$\frac{47}{100}, \frac{1}{2}$$

Central Tendencies

5, 6, 6, 7, 10, 11, 12, 14, 15, 20

Mode: Most common number (6)

- There may be more than one mode
- There is no mode when all numbers are different

Median: Middle number (10.5)

- List numbers in order
- Find the middle number
- If there are two middle numbers, find the average

Mean: Average (10.6)

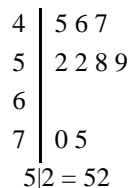
- Add all the numbers together
- Divide by the number of items

Range: Difference between the largest number on list and the smallest number (15)

Stem and Leaf Plots

An organized chart used to arrange data.

70, 52, 58, 45, 59, 52, 75, 47, 46

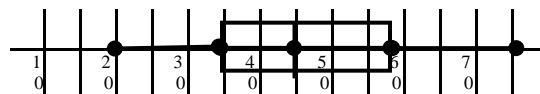


- **Step 1:** Find the least and greatest values
- **Step 2:** Write the stems in a column
- **Step 3:** Arrange the leaves from smallest to largest
- **Step 4:** Write an explanation

Box and Whisker Plots

Shows how data is spread out by dividing it into four groups.

20, 36, 58, 45, 59, 55, 75, 35, 35



- **Step 1:** Draw a line graph with equal intervals
- **Step 2:** Place a dot on the line graph for the smallest and largest values
- **Step 3:** Place a dot on the line graph for the median
- **Step 4:** Place two dots on the line graph, one for the median of the first half of the data and one for the second half
- **Step 5:** Draw a box around the two middle quartiles
- **Step 6:** Place an asterisk (*) on the line graph for an extreme data item (if there is one)